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Relation between $1/f$ noise and frequency-independent loss tangent

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Abstract. A new model has been proposed, which relates the $1/f$ noise in semiconductors to the frequency independent loss tangent in dielectrics. It is demonstrated that this model can explain Hooge's empirical $1/f$ noise relation. The theoretical results are in agreement with experimental results. The model supports the opinion that $1/f$ noise is caused by mobility fluctuations.

1. Introduction

The open-circuit voltage noise S_V of a capacitor with a dielectric without free electrons is given by

$$S_V = 4kT\,\mathrm{Re}(Z) = \frac{4kTR}{1+\omega^2R^2C^2} = \frac{4kT\varepsilon'/(\varepsilon''\omega C)}{1+(\varepsilon'/\varepsilon'')^2} \approx \frac{4kT\tan\delta}{\omega C} \tag{1}$$

with $\mathrm{Re}(Z)$ the real part of the impedance, k the Boltzmann constant, T the temperature, ω the angular frequency and ε' and ε'' the real and imaginary part of the dielectric constant, $\varepsilon_r = \varepsilon' - j\varepsilon''$. The loss tangent is defined by $\tan\delta = \varepsilon''/\varepsilon'$, the capacitance $C = \varepsilon'\varepsilon_0 A/L$, and the loss resistance $R = L/(\omega\varepsilon''\varepsilon_0 A)$ with L the length and A the cross-section of the capacitor. In many dielectrics the loss tangent is usually found to be almost frequency independent from frequencies lower than 10^{-2} Hz to higher than 10^8 Hz, as well as almost temperature independent, and much smaller than one [1]. In these dielectrics both the real part of the dielectric constant ε' and the capacitance C are almost frequency independent. Hence according to equation (1) the voltage noise S_V is inversely proportional to the frequency, just like $1/f$ noise in the conductance G of homogeneous semiconductors [2, 3]. The spectral noise density of the $1/f$ fluctuations in the conductance G can be described by the empirical relation

$$S_G = \alpha G^2/(fN) \tag{2}$$

with α the empirical Hooge parameter, f the frequency ($\omega = 2\pi f$) and N the number of free charge carriers. We can ask ourselves whether there is a relation between $1/f$ noise in semiconductors and frequency independent loss tangent in dielectrics without free electrons.

The first time that $1/f$ noise in electronic devices was related to a constant loss tangent was for the interpretation of $1/f$ current noise in tunnel junctions [4, 5]. Here the transparency is modulated by the thermal noise of the insulator in between the two metal electrodes. According to equation (1) the thermal noise, and thus the transparency

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noise and the current noise, are proportional to $1/f$. Recently, Akiba [6] investigated the $1/f$ noise in silicon p–n junctions at low leakage currents and at temperatures between 110 and 200 K. He interpreted the results in terms of $1/f$ dielectric polarization noise. The loss tangent of the device material, derived from the $1/f$ noise, was found to be in the range of 10^{-4}–2×10^{-4} and constant in frequency.

In materials without free electrons the loss tangent is related to the imaginary part of the dielectric constant ε''. The magnitude of ε'' is determined by the bound electrons. In materials with free electrons, the total loss tangent is determined by both ε'' and the dc electric conductivity σ_c, thus both by the bound electrons and by the free electrons. Here we have

$$\tan\delta_{tot} = 1/(\omega RC) = \varepsilon''/\varepsilon' + \sigma_c/(\omega\varepsilon'\varepsilon_0) \tag{3}$$

with R the resistance determined by the parallel connection of the dielectric loss resistance $L/(\omega\varepsilon''\varepsilon_0 A)$ and the ohmic resistance $L/(\sigma_c A)$. In this paper we try to relate Hooge's $1/f$ noise parameter α to the part of the loss tangent determined by the bound electrons, thus to $\tan\delta = \varepsilon''/\varepsilon'$.

2. Outline of the model

The electric charge density $\rho(\boldsymbol{r})$ of an atom, built up of nuclear and bound electronic charge, can fluctuate. On the condition that the total charge is constant we have

$$\Delta\rho(\boldsymbol{r}, t) = \rho(\boldsymbol{r}, t) - \langle\rho(\boldsymbol{r}, t)\rangle \qquad \text{and} \qquad \int \Delta\rho(\boldsymbol{r}, t)\,\mathrm{d}\boldsymbol{r} = 0. \tag{4}$$

These fluctuations lead to a fluctuating electric dipole. If we put such an atom in between two parallel metal electrodes, then we observe voltage fluctuations across the electrodes. On the other hand, we find that the cross-section for electron scattering by such an atom fluctuates.

A dielectric in between two metal electrodes shows voltage fluctuations according to equation (1). A free electron, moving in such a dielectric, will be scattered by the electric field fluctuations due to $\Delta\rho(\boldsymbol{r}, t)$. These fluctuations lead to fluctuations in the scattering cross-section and thus to fluctuations in the electron mobility with spectral density S_μ. Now the question arises: 'What is the relation between S_μ on the one hand and $S_V = 4kT\varepsilon''/(\varepsilon'\omega C)$ on the other?'.

The organization of the calculations, relating $1/f$ noise and frequency independent loss tangent, is as follows.

- In section 3.1 we shall derive a relation for the fluctuations in the cross-section for electron scattering of an atom due to the fluctuations in the bound electronic charge density of that atom.
- Then we relate the fluctuations in the cross-section to fluctuations in the free path of the free electrons and to fluctuations in their mobility (see section 3.2).
- From the relation between the electronic charge density fluctuations and the dielectric polarization fluctuations we obtain a relation between electronic charge density fluctuations and loss tangent (see section 3.3).
- In section 3.4 we present relations between frequency independent loss tangent, $1/f$ mobility fluctuations and the Hooge $1/f$ noise parameter α.
- In section 3.5 we discuss the uncorrelated behaviour of the mobility fluctuations of the individual electrons. This behaviour leads to the factor $1/N$ in equation (2).

In figure 1 we have presented a flow chart of the organization of the calculations.

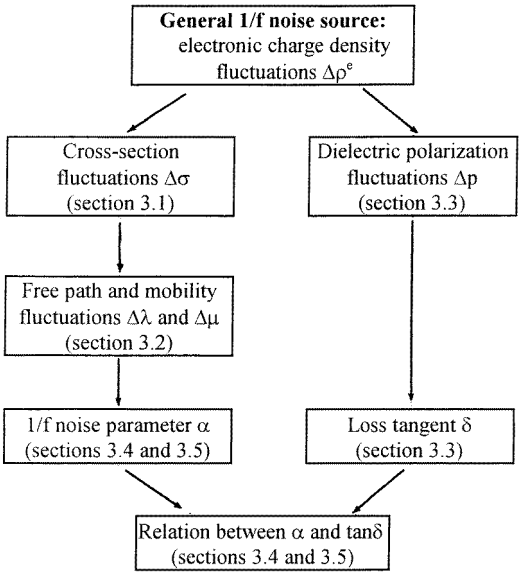

Figure 1. Flow chart of the model calculations.

3. Theoretical approach

3.1. Atomic scattering of electrons

The scattering of electrons by an atom is represented by the scattering amplitude [7]

$$f(\theta) = f(\boldsymbol{k}, \boldsymbol{k}') = -\frac{2\pi m}{h^2} \int \mathrm{e}^{-\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{r}} \phi(\boldsymbol{r})\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\, \mathrm{d}\boldsymbol{r} = \frac{2}{a_0 q K^2} \int \rho(\boldsymbol{r})\, \mathrm{e}^{\mathrm{i}\boldsymbol{K}\cdot\boldsymbol{r}}\, \mathrm{d}\boldsymbol{r} \tag{5}$$

with m the electron mass, h the Planck constant, $\phi(\boldsymbol{r})$ the electrostatic potential of the atom, $\boldsymbol{k}$ and $\boldsymbol{k}'$ the wavevectors of the incident and scattered electron, $\boldsymbol{K} = \boldsymbol{k} - \boldsymbol{k}'$, and $a_0 = h^2\varepsilon_0/\pi q^2 m$ the Bohr radius, with q the elementary charge and ε_0 the permittivity of vacuum. Taking the nucleus at $\boldsymbol{r} = \boldsymbol{0}$, we obtain

$$f(\theta) = \frac{2Z}{a_0 K^2}\left[1 + \frac{1}{Zq}\int \rho^e(\boldsymbol{r})\, \mathrm{e}^{\mathrm{i}\boldsymbol{K}\cdot\boldsymbol{r}}\, \mathrm{d}\boldsymbol{r}\right] = \frac{2Z}{a_0 K^2}\left[1 - F(\boldsymbol{K})\right] \tag{6}$$

where Z is the atomic number, θ the scattering angle, $\rho^e(\boldsymbol{r})$ the electronic charge density and $F(\boldsymbol{K})$ the atomic scattering factor with $F(0) = 1$. Here it should be noted that for the nuclear charge density the integral on the r.h.s. of equation (5) leads to Zq. Fluctuations $\Delta\rho^e(\boldsymbol{r})$ lead to fluctuations in $f(\theta)$ and thus to fluctuations in the differential cross-section $\sigma(\theta) = f(\theta) f^*(\theta)$. The asterisk denotes the complex conjugate. We have

$$\Delta\sigma(\theta) = f(\theta)\Delta f^*(\theta) + f^*(\theta)\Delta f(\theta) \tag{7}$$

and according to equation (6)

$$\Delta f(\theta) = (2/a_0 q K^2) \int \Delta\rho^e(\boldsymbol{r})\, \mathrm{e}^{\mathrm{i}\boldsymbol{K}\cdot\boldsymbol{r}}\, \mathrm{d}\boldsymbol{r}. \tag{8}$$

Assuming a sphere symmetrical charge distribution, then we have [7]

$$F(\boldsymbol{K}) = F(K) = (-4\pi/ZqK) \int_0^\infty r\rho^e(r) \sin(Kr)\, \mathrm{d}r. \tag{9}$$

In this case $F(\boldsymbol{K})$ and thus $f(\theta)$ are real functions. With the help of equations (6)–(9) the fluctuations $\Delta\sigma(\theta)$ are found to be

$$\Delta\sigma(\theta) = 2\,\mathrm{Re}[f(\theta)\Delta f(\theta)] = \frac{8Z[1-F(K)]}{qa_0^2K^4}\int \Delta\rho^e(\boldsymbol{r})\cos(\boldsymbol{K}\cdot\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}. \tag{10}$$

At room temperature the wave vector k of thermal electrons is of the order of $2\pi m v_{th}/h \approx 10^9\ \mathrm{m}^{-1}$. The atomic radius a is of the order of 10^{-10} m. Therefore we can make the approximations

$$Kr \ll 1 \qquad \sin(Kr) \approx Kr - (Kr)^3/6 \qquad \cos(\boldsymbol{K}\cdot\boldsymbol{r}) \approx 1 - (\boldsymbol{K}\cdot\boldsymbol{r})^2/2. \tag{11}$$

With equations (9) and (11) and the relation $\int 4\pi r^2\rho^e(r)\,\mathrm{d}r = -Zq$ we find

$$F(K) \approx 1 + (K^2/6qZ)\int 4\pi r^4\rho^e(r)\,\mathrm{d}r \approx 1 - K^2a^2/6 \tag{12}$$

where a is the atomic radius. Combining equations (10) and (12), using the relation $\int \Delta\rho^e(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} = 0$, and the approximation for $\cos(\boldsymbol{K}\cdot\boldsymbol{r})$ in equation (11), we obtain

$$\Delta\sigma(\theta) = \frac{4Za^2}{3qa_0^2K^2}\int \Delta\rho^e(\boldsymbol{r})\cos(\boldsymbol{K}\cdot\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} \approx -\frac{2Za^2}{3qa_0^2}\int \Delta\rho^e(\boldsymbol{r})\left[\frac{\boldsymbol{K}\cdot\boldsymbol{r}}{K}\right]^2\mathrm{d}\boldsymbol{r}. \tag{13}$$

The fluctuations in the total cross-section are given by [8]

$$\Delta\sigma = \int_0^{2\pi}\int_0^{\pi} \Delta\sigma(\theta)\sin\theta[1-\cos\theta]\,\mathrm{d}\theta\,\mathrm{d}\varphi. \tag{14}$$

To evaluate $\Delta\sigma$ we follow the next procedure. We choose a coordinate system with the x-axis parallel to the wave vector $\boldsymbol{k}$ of the incident electron. The situation is shown in figure 2. We define $\boldsymbol{e} = \boldsymbol{K}/K$ and $\boldsymbol{u} = \boldsymbol{r}/r$, thus $e = u = 1$. The inproduct $\boldsymbol{e}\cdot\boldsymbol{u}$ is given by

$$\boldsymbol{e}\cdot\boldsymbol{u} = e_xu_x + e_yu_y + e_zu_z \tag{15}$$

with

$$\begin{aligned} e_x &= \cos\eta = \sin(\theta/2) & u_x &= \cos\beta \\ e_y &= \sin\eta\cos\varphi = -\cos(\theta/2)\cos\varphi & u_y &= \sin\beta\cos\psi \\ e_z &= \sin\eta\sin\varphi = -\cos(\theta/2)\sin\varphi & u_z &= \sin\beta\sin\psi. \end{aligned} \tag{16}$$

From figure 2 it follows that θ runs from 0 to π, thus the angle π runs from $-\pi/2$ to 0. Consequently we have $\eta = (\theta-\pi)/2$. The angles φ and ψ run from 0 to 2π. The angle β, which is the angle between $\boldsymbol{k}$ and $\boldsymbol{r}$ and thus between x and $\boldsymbol{u}$, runs from 0 to π. With the help of equations (13)–(16) we find

$$\begin{aligned} \Delta\sigma &= -\frac{2Za^2}{3qa_0^2}\iiint \Delta\rho^e(\boldsymbol{r})r^2(\boldsymbol{e}\cdot\boldsymbol{u})^2\sin\theta(1-\cos\theta)\,\mathrm{d}\theta\,\mathrm{d}\varphi\,\mathrm{d}\boldsymbol{r} \\ &= -\frac{4\pi Za^2}{3qa_0^2}\int \Delta\rho^e(\boldsymbol{r})r^2[\cos^2\beta + 1/3]\,\mathrm{d}\boldsymbol{r}. \end{aligned} \tag{17}$$

Equation (17) shows that a charge transfer from the spot $\boldsymbol{r}$ to the spot $-\boldsymbol{r}$ does not give a fluctuation in the cross-section σ. So an electric dipole which jumps between two opposite directions gives $\Delta\sigma = 0$. Such a jump yields $\Delta\rho^e(\boldsymbol{r}) = -\Delta\rho^e(-\boldsymbol{r})$ and $\beta^* = \beta + \pi$, with β^* the angle between $\boldsymbol{k}$ and $-\boldsymbol{r}$, and thus it follows from equation (17) that $\Delta\sigma = 0$.

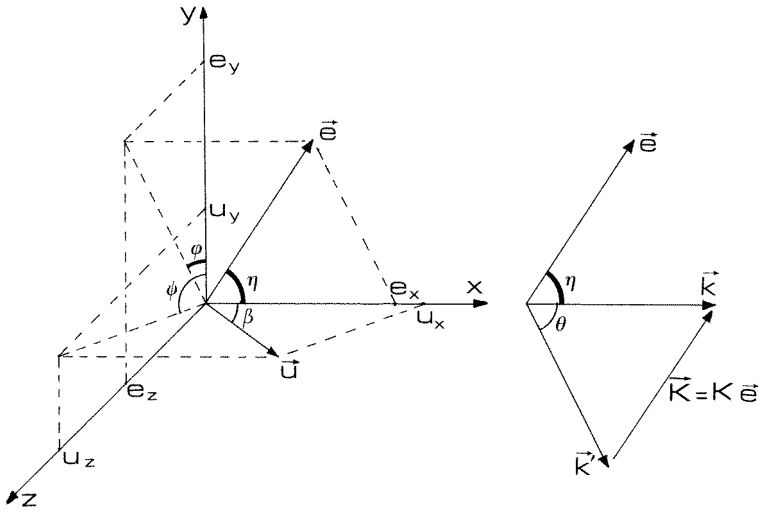

Figure 2. Spherical coordinate system of an atom and the wavevectors $\boldsymbol{k}$ and $\boldsymbol{k}'$ of the incident and scattered electron.

For fluctuations at the edge of the atom ($r = a$), the last part of equation (17) reduces to

$$\Delta\sigma = -\frac{2\pi Z a^4}{3qa_0^2}\int \Delta\rho^e(\boldsymbol{r})[\cos(2\beta) + 5/3]\,\mathrm{d}\boldsymbol{r} = -\frac{2\pi Z a^4}{3qa_0^2}\int \Delta\rho^e(\boldsymbol{r})\cos(2\beta)\,\mathrm{d}\boldsymbol{r} \tag{18}$$

on the condition that the total charge is constant, thus $\int \Delta\rho^e(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} = 0$. Here the relation $\cos^2\beta = 1/2[\cos(2\beta) + 1]$ has been used.

The spectral noise density of the fluctuations $\Delta\sigma$ follows from equation (17)

$$S_\sigma = \frac{16\pi^2 Z^2 a^4}{9q^2 a_0^4}\iint S_{\rho^e}(\boldsymbol{r}, \boldsymbol{r}')r^2 r'^2(\cos^2\beta + 1/3)(\cos^2\beta' + 1/3)\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}' \tag{19}$$

with $S_{\rho^e}(\boldsymbol{r}, \boldsymbol{r}')$ the cross-correlation spectral noise density of the fluctuations $\Delta\rho^e(\boldsymbol{r})$.

3.2. Mobility fluctuations and cross-section fluctuations

Consider a dielectric where the electronic charge density around each atom fluctuates. The fluctuations at different atoms are assumed to be uncorrelated. A free electron moving in this dielectric will have a scattering cross-section per atom σ and a free path λ. Fluctuations $\Delta\sigma$ lead to fluctuations $\Delta\lambda$. An electron travelling the length of a free path passes approximately $p = \lambda/2a$ atoms, where a is the atomic radius. A relative fluctuation $\Delta\sigma/\sigma$ of the scattering cross-section of an atom leads to a relative fluctuation in the free path $\Delta\lambda/\lambda = -(\Delta\sigma/\sigma)/p$. Hence the spectral noise densities of the fluctuations $\Delta\lambda$ and $\Delta\sigma$ are related as

$$S_\lambda/\lambda^2 = [S_\sigma/\sigma^2]/p^2. \tag{20}$$

Since the fluctuations $\Delta\sigma$ of different atoms are uncorrelated, the relative spectral noise density in the free path for a single electron scattered by p atoms is found to be p times the contribution of one atom, thus

$$S_\lambda/\lambda^2 = p(S_\sigma/\sigma^2)/p^2 = 2n^2 a\lambda S_\sigma \tag{21}$$

with $\lambda = 1/(n\sigma)$, and n is the density of atoms, $n \approx 1/(2a)^3$. The mobility μ of a free electron is proportional to its free path λ, thus the relative fluctuations in both quantities are equal. Consequently we have

$$S_\mu/\mu^2 = S_\lambda/\lambda^2 = 2n^2 a\lambda S_\sigma. \tag{22}$$

3.3. Loss tangent and electronic charge density fluctuations

We apply two electrodes to the dielectric. There are no free electrons. Each atom has a dipole $\boldsymbol{p}$ with fluctuations $\Delta\boldsymbol{p}$

$$\boldsymbol{p} = \int \rho^e(\boldsymbol{r})\boldsymbol{r}\,\mathrm{d}\boldsymbol{r} \qquad \text{and} \qquad \Delta\boldsymbol{p} = \int \Delta\rho^e(\boldsymbol{r})\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}. \tag{23}$$

The fluctuations $\Delta\boldsymbol{p}$ lead to voltage fluctuations across the electrodes

$$\Delta V = \Delta p_x/(\varepsilon_r\varepsilon_0 A) \qquad \text{and} \qquad S_V = (\varepsilon_r\varepsilon_0 A)^{-2} S_{p_x} \tag{24}$$

where Δp_x is the component of $\Delta\boldsymbol{p}$ perpendicular to the electrodes. If the dipoles of the $N = nAL$ atoms fluctuate independently, and if the orientation of the dipoles is random, then we obtain, with $\langle\Delta p_x^2\rangle = \langle\Delta\boldsymbol{p}^2\rangle/3$ and $C = \varepsilon_r\varepsilon_0 A/L$,

$$S_V = N(\varepsilon_r\varepsilon_0 A)^{-2} S_{p_x} = [n/(3\varepsilon_r\varepsilon_0 C)]S_p. \tag{25}$$

From equation (23) it follows

$$S_p = \iint S_{\rho^e}(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{r}\cdot\boldsymbol{r}'\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}' \tag{26}$$

with $S_{\rho^e}(\boldsymbol{r},\boldsymbol{r}')$ the cross-correlation spectral noise density of the fluctuations $\Delta\rho^e(\boldsymbol{r})$. Combining equations (1) and (25) yields

$$S_p = 6\varepsilon_r\varepsilon_0 kT\tan\delta/(\pi nf). \tag{27}$$

From equations (26) and (27) we observe that dielectrics with constant $\tan\delta$ yield $S_p \sim 1/f$, $S_{\rho^e}(\boldsymbol{r},\boldsymbol{r}') \sim 1/f$ and consequently $S_\sigma \sim 1/f$ (see equation (19)).

3.4. Relation between Hooge's parameter α and $\tan\delta$

Using equations (22) and (27) and putting $S_\mu/\mu^2 = \alpha/f$, we obtain a relation between α and $\tan\delta$

$$\gamma = \alpha/\tan\delta = \frac{12}{\pi} na\lambda\varepsilon_r\varepsilon_0 kT S_\sigma/S_p. \tag{28}$$

Using equations (19) and (26) and taking $n = 1/(2a)^3$ the factor γ becomes

$$\gamma = \alpha/\tan\delta = \frac{8\pi Z^2 a^2\lambda\varepsilon_r\varepsilon_0 kT}{3q^2 a_0^4} r_0^2 \tag{29}$$

where the quantity r_0^2 is given by

$$r_0^2 = \frac{\iint S_{\rho^e}(\boldsymbol{r},\boldsymbol{r}')r^2 r'^2(\cos^2\beta + 1/3)(\cos^2\beta' + 1/3)\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'}{\iint S_{\rho^e}(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{r}\cdot\boldsymbol{r}'\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'} \tag{30}$$

To evaluate r_0^2, we have to know how the fluctuations $\Delta\rho^e(\boldsymbol{r})$ continue. There are several possibilities.

If the fluctuations $\Delta\rho^e(\boldsymbol{r})$ are random, and spatially uncorrelated, we have the cross-correlation spectral noise density of the charge fluctuations to be

$$S_{\rho^e}(\boldsymbol{r},\boldsymbol{r}') = H(\boldsymbol{r},\boldsymbol{r}')[\delta(\boldsymbol{r}-\boldsymbol{r}') - \Omega_v^{-1}] \tag{31}$$

with δ the Dirac delta function, and $\Omega_v = 4\pi a^3/3$ the volume of an atom. The quantity H represents the magnitude of the noise. The term Ω_v in equation (31) stems from the constraint $\int \Delta\rho^e(\boldsymbol{r}, t)\,\mathrm{d}\boldsymbol{r} = 0$. The proof can be found in appendix C in [9]. Equation (31) is analogous to equation (C8) in [9].

For a sphere symmetrical charge distribution and for fluctuations occurring at the edge of the atom ($r = a$), equation (31) becomes

$$S_{\rho^e}(\boldsymbol{r}, \boldsymbol{r}') = H_0[\delta(\boldsymbol{r} - \boldsymbol{r}') - \Omega_A^{-1}] \tag{32}$$

with $|\boldsymbol{r}| = |\boldsymbol{r}'| = a$ and $\Omega_A = 4\pi a^2$. Substituting equation (32) into equation (30) we find the quantity r_0^2 to be

$$\begin{aligned} r_0^2 &= \frac{\int H_0 a^4[\cos^2\beta + 1/3]^2\,\mathrm{d}\boldsymbol{r} - \Omega_A^{-1}\iint H_0 a^4[\cos^2\beta + 1/3][\cos^2\beta' + 1/3]\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'}{\int H_0 a^2\,\mathrm{d}\boldsymbol{r} - \Omega_A^{-1}\iint H_0 \boldsymbol{r}\cdot\boldsymbol{r}'\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'} \\ &= a^2[\langle(\cos^2\beta + 1/3)^2\rangle - \langle(\cos^2\beta + 1/3)\rangle^2] = a^2[\langle\cos^4\beta\rangle - \langle\cos^2\beta\rangle^2] \\ &= a^2/8. \end{aligned} \tag{33}$$

Since the inproduct $\boldsymbol{r}\cdot\boldsymbol{r}'$ can have all values in between $-a^2$ and a^2, the second integral in the denominator of equation (33) equals zero. The average $\langle\ \rangle$ is taken over the angle β between $\boldsymbol{r}$ and $\boldsymbol{k}$. From equations (29) and (33) we find

$$\alpha = \gamma\tan\delta \approx [\pi Z^2\lambda\varepsilon_r\varepsilon_0 kT(a/a_0)^4/3q^2]\tan\delta. \tag{34}$$

For fluctuations $\Delta\rho^e(\boldsymbol{r}, t)$, which are not uncorrelated with respect to the position $\boldsymbol{r}$ and which are not caused by dipoles jumping between opposite directions, we expect to have γ values that have an order of magnitude in accordance with equation (34). Two examples are given below.

Let us assume that the fluctuations occur at the edge of the atom, thus $|\boldsymbol{r}| = |\boldsymbol{r}'| = a$. For a charge displacement Q from spot $\boldsymbol{r}$ to spot $\boldsymbol{r}'$, we find with the help of equations (18) and (23)

$$\begin{aligned} \Delta\sigma &= \frac{-2\pi Z a^4 Q}{3q a_0^2}[\cos(2\beta') - \cos(2\beta)] \\ \Delta\boldsymbol{p} &= -Q(\boldsymbol{r} - \boldsymbol{r}'). \end{aligned} \tag{35}$$

For $\beta' = \beta$ and for $\beta' = \pi - \beta$ we obtain $\Delta\sigma = 0$ and thus $\gamma = 0$. If β' and β are randomly distributed between 0 and π, equation (35) leads to $\langle\Delta\sigma\rangle = 0$, $\langle\Delta\boldsymbol{p}\rangle = 0$, $\langle\Delta\sigma^2\rangle = 4\pi^2 Z^2 a^8 Q^2/(9q^2 a_0^4)$ and $\langle\Delta\boldsymbol{p}^2\rangle = 2Q^2a^2$. With the help of equation (28) we find the factor γ to be

$$\gamma = \frac{12}{\pi} na\lambda\varepsilon_r\varepsilon_0 kT\langle\Delta\sigma^2\rangle/\langle\Delta\boldsymbol{p}^2\rangle = \pi Z^2\lambda\varepsilon_r\varepsilon_0 kT(a/a_0)^4/3q^2. \tag{36}$$

This result is the same as that for random fluctuations at the edge of the atom (see equation (34)). Another approach is to assume a charge displacement Q from spot $\boldsymbol{r}$ to spot $\boldsymbol{r}' = \zeta\boldsymbol{r}$, with $|\boldsymbol{r}| \approx |\boldsymbol{r}'| \approx a$ and $\zeta \approx 1$. Here equations (17) and (23) lead to

$$\begin{aligned} \Delta\sigma &= \frac{-4\pi Z a^4 Q}{3q a_0^2}(\zeta^2 - 1)(\cos^2\beta + 1/3) \\ \Delta\boldsymbol{p} &= (\zeta - 1)Q\boldsymbol{r}. \end{aligned} \tag{37}$$

Now we obtain $\langle\Delta\sigma^2\rangle/\langle\Delta\boldsymbol{p}^2\rangle \approx 6\pi^2 Z^2 a^8/q^2 a_0^4$ and thus with equation (36)

$$\gamma = 9\pi Z^2\lambda\varepsilon_r\varepsilon_0 kT(a/a_0)^4/q^2. \tag{38}$$

Here the numerical factor is a factor of 27 larger than in equations (34) and (36).

3.5. Factor 1/N

For a single electron moving in a dielectric we obtain $S_\mu/\mu^2 = \alpha/f$ with $\alpha = \gamma \tan\delta$. What happens if N electrons move criss-cross through the dielectric? For fluctuations at the edge of the atoms we have according to equation (18)

$$\Delta\sigma \sim \int \Delta\rho^e(\boldsymbol{r}) \cos(2\beta)\, \mathrm{d}\boldsymbol{r}. \tag{39}$$

The fluctuation $\Delta\sigma$ depends on β, thus on the direction of $\boldsymbol{k}$ of the incident electron. For a given position $\boldsymbol{r}$, the average of $\cos(2\beta)$ over all directions of $\boldsymbol{k}$ is $\langle\cos(2\beta)\rangle_k = 0$, and thus $\langle\Delta\sigma\rangle_k = 0$. Since the N electrons move independently of each other, we have $\langle \boldsymbol{k}_i \cdot \boldsymbol{k}_j \rangle = 0$ for $i \neq j$ and $i,\ j = 1$ to N. Consequently, we find the fluctuation $\Delta\sigma_i$ for the electron with $\boldsymbol{k}_i$ to be uncorrelated with $\Delta\sigma_j$ for $j \neq i$, thus $\langle \Delta\sigma_i \cdot \Delta\sigma_j \rangle = 0$ for $i \neq j$. As a result, the fluctuations in the free path of the individual electrons are uncorrelated. For the relative conductance noise we then obtain [10]

$$S_G/G^2 = S_{\mu^*}/\mu^{*2} \approx \gamma \tan\delta/fN \approx \alpha/fN \tag{40}$$

where μ^* is the average mobility of the N electrons, i.e. $\mu^* = (1/N)\sum_{i=1}^{N} \mu_i$, and $G = q\mu^* N/L^2$.

In the general case we have according to equation (17)

$$\langle \cos^2\beta + 1/3 \rangle_k = 5/6 \tag{41}$$

so that the average $\langle\Delta\sigma\rangle_k$ is given by

$$\langle\Delta\sigma\rangle_k = \frac{-10\pi Z a^2}{9qa_0^2} \int \Delta\rho^e(\boldsymbol{r}) r^2\, \mathrm{d}\boldsymbol{r} \sim \int \Delta\rho^e(\boldsymbol{r})(r^2 - a^2)\, \mathrm{d}\boldsymbol{r}. \tag{42}$$

This average is small for fluctuations $\Delta\rho^e(\boldsymbol{r})$ at positions $\boldsymbol{r}$ with $|\boldsymbol{r}| \approx a$. Therefore, equation (40) prevails if the fluctuations in the electronic charge density occur around the edge of the atoms.

3.6. Remarks on the model

Several remarks have to be made concerning the model proposed here. In the model a number of assumptions have been made. Without these assumptions it was not possible to achieve the analytical results presented in this work. The most important assumptions are:

(i) The free electrons are scattered in an elastic way by each atom of the lattice. These atoms are assumed to be immobile in the lattice in spite of their thermal vibrations. The scattering is a result of the Coulomb interaction between an atom and a free electron.

(ii) The electronic charge density of each atom freely fluctuates in an uncorrelated way from that of the other atoms, even of the neighbours.

With respect to assumption (i), it should be noted that the calculations are inspired by the Conwell–Weiskopf approximation on electron scattering on ionized impurity atoms [8]. They assumed that the free electrons are scattered by each atom with an electric charge, and that the atoms are assumed to be immobile in the lattice. Moreover, they

assumed that a free electron is scattered at any time only by the centre to which it is closest. The same assumptions are made in the model presented here. Just as in the Conwell–Weiskopf model, thermal vibrations of the atoms are not taken into account. These vibrations are very fast and do not lead to low-frequency noise, because of the very short correlation time related to these vibrations. Regarding assumption (ii), we have to remark that the atoms in a dielectric can have free electric dipoles and/or induced dipoles. The orientations of these dipoles are often assumed to be more or less uncorrelated of each other, so that electronic charge density fluctuations (dipole fluctuations) of the lattice atoms can be ascribed locally to each atom. We have to realize that this is an approximation. In fact there will be an interaction between neighbours.

Finally, the question remains of what is the physical interpretation of the characteristic $1/f$ fluctuations in the electronic charge density and consequently in the electric field. This question is still open, and not solved here. The object of this paper is to demonstrate that $1/f$ noise in the conductance of semiconductors may have the same physical origin as the frequency independent loss tangent in dielectrics.

4. Comparison of $\alpha = \gamma \tan \delta$ with experimental data

Equation (5) holds also for scattering of free electrons in a semiconductor, provided that we replace m by the effective mass m^* of the free electrons, the permittivity ε_0 by $\varepsilon_r\varepsilon_0$ and thus a_0 by $a_0\varepsilon_r m/m^*$. Applying equations (34), (36) and (38) to semiconductors, we have to replace a_0 by $a_0\varepsilon_r m/m^*$, thus

$$\gamma = \alpha/\tan\delta \approx Z^2\lambda\varepsilon_r\varepsilon_0 kT(m^*a/m\varepsilon_r a_0)^4/q^2. \tag{43}$$

Now we shall present numerical values for the factor γ and we shall compare these γ values with the ratio of experimental data of the Hooge parameter α and the frequency independent loss factor $\tan\delta$.

We have calculated the factor γ for a number of elementary semiconductors, both n type and p type, where the free carrier mobilities are determined by lattice scattering. The values for the free path λ have been derived from the relation between free carrier mobility μ and free path λ

$$\mu = q\tau/m^* = q\lambda/(m^*v_{th}) \tag{44}$$

where τ is the collision time of the free carriers, and $v_{th} = (3kT/m^*)^{1/2}$ their thermal velocity. The data on dielectric constant ε_r, effective mass m^* and mobility μ are obtained for the most part from [11]. From the results presented in table 1 it follows that the factor γ is in the range of 10^{-2} to 10^{-1}. The factor γ of all these semiconductors is of the same order of magnitude, which is not so surprising. Semiconductors with higher atomic numbers generally have lower effective free carrier masses, higher carrier mobilities and somewhat higher atomic radii. Apparently these effects counterbalance each other, so that the magnitude of γ does not vary strongly. It should be noted that for lattice scattering the free path λ is roughly inversely proportional to the temperature T, hence the factor γ should be almost temperature independent.

In an extensive review paper [3] Hooge presented a lot of experimental values for α taken from literature. He included only results obtained from homogeneous semiconductor samples at room temperature. For materials where lattice scattering dominates, α values are in the range of 10^{-6} to 10^{-3}. His review shows that 10^{-4} is the order of magnitude of the average value α for all the materials considered, i.e. Si, GaAs, Ge, CdHgTe, InSb

Table 1. Model parameters and γ values for diamond (C), silicon (Si), germanium (Ge) and tin (Sn).

Material	Atomic number Z	Dielectric constant ε_r	Atomic radius a (nm)	Effective mass m^*/m	Mobility at 300 K μ (m^2 V^{-1} s^{-1})	Free path at 300 K λ (μm)	Factor γ at 300 K
n-C	6	5.7	0.091	0.57	0.22	0.11	0.030
p-C	6	5.7	0.091	0.80	0.18	0.11	0.110
n-Si	14	11.7	0.146	0.36	0.14	0.057	0.010
p-Si	14	11.7	0.146	0.49	0.05	0.023	0.012
n-Ge	32	16.2	0.152	0.22	0.39	0.12	0.007
p-Ge	32	16.2	0.152	0.33	0.19	0.072	0.020
p-Sn	50	24	0.172	0.30	0.24	0.087	0.020

and InP. In view of the wide range of observed α values, no significant differences in α values of different materials are observed. Hooge and Tacano [12] have reviewed the literature on $1/f$ noise in n-GaAs. For lattice scattering and in the temperature range 77–300 K they found the α values to be in the range of 10^{-5} to 10^{-4}. Clevers [13] has investigated the $1/f$ noise in both n- and p-type Si with various dopant concentrations in the temperature range from 77 K to 300 K. He observed also a wide range of α values, between 10^{-6} and 10^{-3} at 300 K and between 10^{-7} and 10^{-3} at 77 K. According to Clevers the broad scattering in experimental data is possibly related to the manufacturing process. According to Hooge [3] there is evidence that the value of α is increased by lattice imperfections. This has led to the rule: the lower the α value the better the crystal. From the above the conclusion is justified that as a general rule high quality semiconductor crystals have α values not larger than 10^{-5}.

A review of dielectric properties of a wide range of solids has been given by Jonscher [1]. It was observed that the dielectric response functions depart strongly from the Debye function for a large number of materials. In particular, the frequency dependence of the dielectric losses follows the empirical relation

$$\tan\delta \sim \varepsilon''(f) \sim f^{n-1} \tag{45}$$

with n a positive number close to unity. Equation (45) holds for a wide frequency range, from frequencies lower than 10^{-2} to over 10^8 Hz. A review of the empirical description of such dielectric phenomena has been given by Böttcher and Bordewijk [14]. More experimental data on the frequency independent loss tangent can be found in [15–18]. The frequency independent $\tan\delta$ is found to be nearly temperature independent. For inorganic pure single crystals the value for $\tan\delta$ is often found to be of the order of magnitude of 10^{-4}. Recently, Akiba [6] found a $\tan\delta$ value of the order of 10^{-4} in silicon p–n junctions.

In view of the broad scattering in experimental data both on the Hooge parameter α and on the frequency independent loss tangent and taking into account the number of assumptions made in our model (see section 3.6), we cannot expect a perfect agreement between experimental data of the ratio $\alpha/\tan\delta$ and the theoretical expression for $\gamma = \alpha/\tan\delta$ given by equation (43). Nevertheless we can state that the order of magnitude of the experimental values for $\gamma = \alpha/\tan\delta \approx 0.1$ agrees with the calculated values of 10^{-2}–10^{-1}.

5. Discussion and conclusion

The cross-correlation spectral density of $1/f$ fluctuations in the conductivity of semiconductors can be written as [19]

$$S_{\sigma_c}(\boldsymbol{r}, \boldsymbol{r}', E, E', f) = \frac{\alpha \sigma_c^2(\boldsymbol{r}, E)}{f n(\boldsymbol{r}, E)} \delta(\boldsymbol{r} - \boldsymbol{r}') \delta(E - E') \tag{46}$$

with E the kinetic energy of the free carriers, $n(\boldsymbol{r}, E)$ the density of the carriers with energy E at the spot $\boldsymbol{r}$, $\sigma_c(\boldsymbol{r}, E)$ the conductivity of carriers with energy E at the spot $\boldsymbol{r}$, α the Hooge parameter and δ the Dirac delta function.

In section 3.5 we have shown that the contributions of the individual electrons to the fluctuations are uncorrelated. So we may conclude that electrons with different energies E also contribute uncorrelatedly, hence the term $\delta(E - E')$ in equation (46). The fluctuations $\Delta\rho^e(\boldsymbol{r}, t)$ are assumed to be uncorrelated at different atoms. Therefore the conductivity fluctuations $\Delta\sigma_c$ are spatially uncorrelated at distances larger than the atomic distances, hence the term $\delta(\boldsymbol{r} - \boldsymbol{r}')$ in equation (46). According to equations (34), (36) and (38) the $1/f$ noise parameter α is independent of the energy E of the electrons, provided that λ is independent of E, which is the case in lattice scattering. This is in agreement with the experimental results of $1/f$ noise in thermo-e.m.f. [20].

From equation (43) it follows that α is proportional to the mean free path λ. If the mean free path is reduced by imperfections such as impurities and lattice defects, then we have

$$\lambda^* = \lambda\lambda_i/(\lambda + \lambda_i) \tag{47}$$

where λ is the free path without imperfections, and λ_i is the free path for imperfection scattering only. The free path λ_i is related to the distance between the imperfections and is assumed to be noiseless. The fluctuations in λ^* due to fluctuations in λ are given by

$$\Delta\lambda^* = [\lambda_i/(\lambda + \lambda_i)]^2 \Delta\lambda = (\lambda^*/\lambda)^2 \Delta\lambda. \tag{48}$$

Since $\lambda^* \sim \mu$, $\lambda \sim \mu_{latt}$ and $\lambda_i \sim \mu_{imp}$ we obtain

$$\frac{S_{\lambda^*}}{\lambda^{*2}} = \frac{S_\mu}{\mu^2} = \left(\frac{\lambda^*}{\lambda}\right)^2 \frac{S_\lambda}{\lambda^2} = \left(\frac{\mu}{\mu_{latt}}\right)^2 \frac{S_{\mu_{latt}}}{\mu_{latt}^2}. \tag{49}$$

Equation (49) is in agreement with experimental results [2].

In view of the present results it can be concluded that there is a relation between $1/f$ noise in semiconductors and the frequency independent loss tangent in dielectrics. They have the same physical origin: $1/f$ fluctuations in the electronic charge density of an atom.

It is obvious that the present model is compatible with the mobility fluctuation model, but not with the McWhorter model for number fluctuations. The model predicts values for α of the right order of magnitude.

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